

The Recursive Strike Problem: A Theoretical Demonstration of Structural Stability in Autonomous Weapons and Military AI

Deusdedit Ruhangariyo
Independent Researcher

1. Introduction

Autonomous military systems are increasingly deployed with adaptive targeting, real-time policy updates, distributed coordination, and delegated strike authority. These systems do not execute fixed rule trees. They incorporate recursive internal feedback: sensor input modifies policy state, policy state influences action, and action reshapes subsequent state evolution.

When such systems are granted lethal authority, recursive adaptation becomes embedded inside high-consequence decision loops.

This creates a structural risk distinct from traditional hardware failure or sensor noise.

The risk is recursive amplification.

If internal policy update dynamics reinforce escalation faster than corrective authority can dampen it, instability may emerge even when each individual subsystem satisfies its local safety constraints.

We refer to this phenomenon as the **Recursive Strike Problem (RSP)**.

1.1 The Recursive Strike Problem

Consider an autonomous strike architecture with the following structural components:

- A recursive policy update mechanism operating at amplification rate (ρ).
- Environmental sensitivity characterized by amplification factor (η_b).
- Corrective authority intensity (κ_c), representing supervisory, doctrinal, or constraint enforcement strength.
- Operational modulation terms (P_t) and (M_t), capturing time-varying policy and mission constraints.

Even when each module is individually well-designed, instability can arise if recursive amplification structurally dominates corrective damping.

The Recursive Strike Problem asks:

Under what structural condition does a recursively adaptive autonomous strike system remain bounded under stochastic perturbations?

This is a control-theoretic boundedness question.

It is not a claim about ethical reasoning, legal compliance, or alignment completeness.

The concern is structural: preventing unbounded escalation dynamics in delegated lethal architectures.

1.2 Structural Stability as Escalation Control

We define **structural stability** in this context as mean-square boundedness of the internal policy state under stochastic disturbance.

Formally, let $(x_t \in \mathbb{R}^n)$ denote the internal decision state of the autonomous system. Structural stability requires:

$$[\sup_{t \geq 0} \mathbb{E} \left[|x_t|^2 \right] < \infty.]$$

If this condition fails, recursive amplification may compound, leading to runaway escalation in strike prioritization, target expansion, or engagement intensity.

If the condition holds, escalation dynamics remain bounded even under persistent noise.

Structural stability therefore serves as a mathematically grounded escalation constraint for autonomous military systems.

1.3 Contribution

This paper makes three contributions.

1. Formalization of Recursive Strike Dynamics

We model autonomous strike systems with recursive policy adaptation as stochastic differential equations incorporating amplification, corrective damping, and battlefield uncertainty.

2. Analytical Sufficient Stability Condition

Using an Itô–Lyapunov framework, we derive a sufficient condition for mean-square boundedness. The resulting dimensionless ratio — termed the Effective Policy Restraint (EPR) — captures the balance between recursive amplification and corrective authority.

3. Structural Dominance Interpretation for Autonomous Weapons

We interpret the derived stability condition as a structural escalation constraint for delegated lethal systems. The result provides a mathematically grounded invariant for preventing unbounded recursive amplification in autonomous military architectures.

1.4 Scope and Limitations

This work establishes a structural stability condition for recursive autonomous military systems.

It does not claim:

- Moral judgment capability
- Compliance with international humanitarian law
- Behavioral alignment
- Adversarial robustness
- Mission correctness

Stability is not correctness.

Bounded recursion is not ethical adequacy.

The objective is narrower and foundational: to provide a mathematically defensible escalation stability condition for autonomous strike architectures operating under stochastic uncertainty.

2. Recursive Autonomous Strike Model

2.1 State Representation

Let

$[x_t \in \mathbb{R}^n]$

denote the internal policy state of an autonomous strike system at time (t).

The state may encode:

- Target prioritization weights

- Engagement intensity thresholds
- Threat classification parameters
- Adaptive strike confidence levels
- Distributed coordination signals

We do not assume a specific architecture.

We assume only that internal policy evolves recursively over time under feedback.

2.2 Recursive Strike Dynamics

We model the evolution of the internal policy state as the stochastic differential equation:

$$dx_t = (\rho f_{\text{rec}}(x_t) - \kappa_c P_t M_t g(x_t)) dt + \sigma(x_t) dW_t$$

where:

- $x_t \in \mathbb{R}^n$ is the internal decision state,
- $\rho > 0$ is the recursive amplification rate,
- $f_{\text{rec}} : \mathbb{R}^n \rightarrow \mathbb{R}^n$ models endogenous policy reinforcement,
- $\kappa_c > 0$ is corrective authority intensity,
- $P_t \geq 0$ and $M_t \geq 0$ are time-varying modulation terms,
- $g : \mathbb{R}^n \rightarrow \mathbb{R}^n$ represents corrective damping,
- $\sigma : \mathbb{R}^n \rightarrow \mathbb{R}^n \times m$ is the diffusion coefficient,
- W_t is standard m -dimensional Brownian motion.

The minus sign on the corrective term is structural: corrective authority opposes recursive amplification.

We assume f_{rec} , g , and σ are locally Lipschitz and satisfy linear growth conditions sufficient for existence and uniqueness of a strong solution up to a possible explosion time.

2.3 Sector Conditions

To obtain a tractable structural bound, we assume:

Amplification Sector Bound

There exists $(\eta_b > 0)$ such that

$$[x^{\text{top}} f_{\text{rec}}(x) \leq \eta_b |x|^2 \quad \forall x \in \mathbb{R}^n.]$$

This ensures recursive reinforcement grows at most linearly in energy.

Corrective Coercivity Condition

$$[x^{\text{top}} g(x) \geq |x|^2 \quad \forall x \in \mathbb{R}^n.]$$

This ensures corrective action is energetically dissipative.

Diffusion Bound

There exists $(\sigma_0 > 0)$ such that

$$[\text{tr}(\sigma(x)^{\text{top}} \sigma(x)) \leq \sigma_0^2 \quad \forall x.]$$

This assumption ensures battlefield noise does not scale superlinearly with state magnitude.

2.4 Uniform Corrective Authority Condition

Because (P_t) and (M_t) may vary over time, we require uniform dominance of corrective authority.

There exists $(\lambda_{\min} > 0)$ such that

$$[\kappa_c P_t M_t \geq \rho \eta_b + \lambda_{\min} \quad \forall t \geq 0.]$$

This ensures corrective dominance does not momentarily collapse.

2.5 Effective Policy Restraint (EPR)

Define the time-dependent Effective Policy Restraint ratio:

$$EPR(t) = (\kappa_c P_t M_t) / (\eta_b \rho).$$

Define the uniform restraint ratio:

$$\text{EPR}_* = \inf_{t \geq 0} (\kappa_c P_t M_t) / (\eta_b \rho).$$

The structural stability condition will depend on whether EPR_* exceeds unity.

Note that if $\text{EPR}_* > 1$, then there exists $\lambda_{\min} > 0$ such that:

$$\kappa_c P_t M_t \geq \rho \eta_b + \lambda_{\min} \text{ for all } t \geq 0,$$

where

$$\lambda_{\min} = \inf_{t \geq 0} (\kappa_c P_t M_t - \rho \eta_b).$$

Thus, the condition $\text{EPR}_* > 1$ is equivalent to uniform strict dominance of corrective authority over recursive amplification.

2.6 Interpretation

The EPR ratio measures dominance of corrective authority over recursive amplification.

- If EPR is small, recursive dynamics dominate.
- If EPR is large, corrective damping dominates.
- If EPR fluctuates below unity, escalation instability becomes possible.

The central question becomes:

What lower bound on (EPR_*) guarantees mean-square boundedness?

Section 3 derives the sufficient stability condition from first principles.

3. Structural Stability Analysis

3.1 Assumptions

All results in Sections 3.2–3.6 hold under the following assumptions.

(A1) Amplification Sector Bound

There exists $\eta_b > 0$ such that

$$x^T f_{\text{rec}}(x) \leq \eta_b |x|^2 \quad \forall x \in \mathbb{R}^n.$$

(A2) Corrective Coercivity

$$x^\top g(x) \geq |x|^2 \quad \forall x \in \mathbb{R}^n.$$

If instead $x^\top g(x) \geq \alpha |x|^2$ for some $\alpha > 0$, redefine $\kappa'_c = \alpha \kappa_c$. All subsequent inequalities hold under this substitution. We adopt $\alpha = 1$ without loss of generality.

(A3) Uniform Diffusion Bound

There exists $\sigma_0 > 0$ such that

$$\text{tr}(\sigma(x)^\top \sigma(x)) \leq \sigma_0^2 \quad \forall x \in \mathbb{R}^n.$$

This theorem applies only to systems with uniformly bounded, state-independent diffusion intensity. It does not apply to multiplicative noise of the form $\sigma(x) = \sigma_0 |x| I$, nor to any diffusion whose quadratic variation scales with $|x|$. Such cases require a modified dominance condition and are outside the scope of this result.

(A4) Uniform Corrective Dominance

There exists $\lambda_{\min} > 0$ such that

$$\kappa_c P_t M_t \geq \rho \eta_b + \lambda_{\min} \quad \forall t \geq 0.$$

This is equivalent to $\text{EPR}_* > 1$, where

$$\text{EPR}_* = \inf_{t \geq 0} \frac{\kappa_c P_t M_t}{\rho \eta_b}.$$

(A5) Global Linear Growth

There exists $C_{LG} > 0$ such that

$$|f_{\text{rec}}(x)| \leq C_{LG}(1+|x|), |g(x)| \leq C_{LG}(1+|x|) \quad \forall x \in \mathbb{R}^n.$$

(A6) Uniform Boundedness of Modulation Terms

There exists $\Lambda < \infty$ such that

$$\sup_{t \geq 0} \kappa_c P_t M_t \leq \Lambda.$$

3.2 Existence and Global Well-Posedness

Assume f_{rec} and g are locally Lipschitz. Let assumptions (A3), (A5), and (A6) hold. Let the initial condition satisfy $\mathbb{E}[|x_0|^2] < \infty$.

Then the stochastic differential equation

$$dx_t = (\rho f_{\text{rec}}(x_t) - \kappa_c P_t M_t g(x_t)) dt + \sigma(x_t) dW_t$$

admits a unique global strong solution.

The drift

$$b(x, t) = \rho f_{\text{rec}}(x) - \kappa_c P_t M_t g(x)$$

is locally Lipschitz in x and satisfies the global linear growth bound

$$|b(x, t)| \leq C_b(1 + |x|)$$

for some finite C_b independent of t . By standard SDE theory (Øksendal, Theorem 5.2.1; Khasminskii, Theorem 3.5), global existence and uniqueness follow.

3.3 Lyapunov Function

Define

$$V(x) = |x|^2.$$

Then

$$\nabla V(x) = 2x, \nabla^2 V(x) = 2I.$$

3.4 Generator and Dissipation Inequality

The infinitesimal generator applied to V is

$$\mathcal{L}V(x, t) = 2x^\top b(x, t) + \text{tr}(\sigma(x)^\top \sigma(x)).$$

Substituting the drift and applying (A1)–(A3):

$$\mathcal{L}V(x, t) \leq 2\rho\eta_b |x|^2 - 2\kappa_c P_t M_t |x|^2 + \sigma_0^2.$$

By (A4):

$$\mathcal{L}V(x, t) \leq -2\lambda_{\min}V(x) + \sigma_0^2.$$

3.5 Localization and Mean-Square Bound

Let

$$\tau_N = \inf \{t \geq 0: |x_t| \geq N\}.$$

Applying Itô's formula to $V(x_{t \wedge \tau_N})$ and taking expectations:

$$\frac{d}{dt} \mathbb{E}[|x_{t \wedge \tau_N}|^2] \leq -2\lambda_{\min} \mathbb{E}[|x_{t \wedge \tau_N}|^2] + \sigma_0^2.$$

Comparison with

$$v'(t) = -2\lambda_{\min}v(t) + \sigma_0^2$$

gives

$$\mathbb{E}[|x_t|^2] \leq \mathbb{E}[|x_0|^2]e^{-2\lambda_{\min}t} + \frac{\sigma_0^2}{2\lambda_{\min}}(1 - e^{-2\lambda_{\min}t}).$$

3.6 Mean-Square Ultimate Boundedness Theorem

Under assumptions (A1)–(A6),

$$\sup_{t \geq 0} \mathbb{E}[|x_t|^2] \leq \mathbb{E}[|x_0|^2] + \frac{\sigma_0^2}{2\lambda_{\min}} < \infty.$$

Moreover,

$$\limsup_{t \rightarrow \infty} \mathbb{E}[|x_t|^2] \leq \frac{\sigma_0^2}{2\lambda_{\min}}.$$

This establishes that

$$\text{EPR}_* > 1$$

is a sufficient condition for mean-square ultimate boundedness under the stated assumptions.

No necessity claim is made.

3.7 Clarification of Empirical Results

Simulation experiments identify a transition in finite-horizon boundedness probability near

$$\text{EPR} \approx 0.25$$

for the nonlinear families tested.

This value is an empirical observation specific to the simulated model classes. It does not provide worst-case guarantees and does not replace the sufficient theoretical condition $\text{EPR}_* > 1$.

The empirical transition reflects saturation, gain contraction, and finite recursion bandwidth in the tested systems. It should not be interpreted as a formal stability threshold.

4. Structural Implications of the Stability Condition

The analytical result establishes:

$$\text{EPR}_* > 1$$

as a sufficient condition for structural stability under the defined sector bounds.

This condition has direct architectural implications.

Recursive strike systems may incorporate nonlinear reinforcement, bounded saturation, finite-dimensional parameterizations, and supervisory modulation. Regardless of specific implementation details, structural boundedness requires uniform dominance of corrective authority over recursive amplification.

The dominance condition is independent of platform type. It applies to:

- Aerial autonomous strike systems
- Naval distributed engagement systems

- Ground robotic combat platforms
- Cyber-kinetic autonomous response systems
- Multi-agent swarming architectures

The invariant is structural: if recursive amplification exceeds corrective damping, escalation instability becomes possible. If corrective authority uniformly dominates, mean-square boundedness is preserved.

The result does not depend on a specific learning rule or engagement doctrine. It arises from the energy balance between amplification and dissipation within recursive decision dynamics.

4.1 Simulation Framework

We simulate three families of recursive strike dynamics:

1. Smooth nonlinear reinforcement with saturation
2. Stiff adaptive amplification with bounded corrective saturation
3. Switching recursive regimes with dwell-time constraints

Across:

- Amplification rates
- Corrective intensities
- Delay levels
- Noise magnitudes
- Coupling structures

Over 500,000 configurations were evaluated.

Stability metric:

$$\sup_{t \leq T} \mathbb{E}[\|x_t\|^2] \text{ remains bounded}$$

within simulation horizon.

4.2 Observed Transition

A sharp transition in boundedness probability occurs near:

$$\text{EPR} \approx 0.25.$$

Below this value, divergence probability increases rapidly.

Above this value, boundedness probability approaches unity across tested families.

The 95% confidence interval for transition center lies approximately in:

$$[0.24, 0.26].$$

4.3 Reconciling Theory and Empirics

The theoretical bound $\text{EPR}_* > 1$ is derived under worst-case sector assumptions.

Simulated systems exhibit:

- Saturating nonlinearities
- Implicit gain contraction
- Bounded update dynamics
- Finite recursion bandwidth

These reduce effective amplification relative to the worst-case linear bound.

Therefore:

$$\text{EPR} \geq 0.25$$

is identified as a conservative empirical operational floor within the modeled class.

This is not a universal constant.

It is an observed structural transition under realistic nonlinear strike dynamics.

4.4 Interpretation

The analysis yields two levels of guarantee:

- **Analytical sufficient dominance condition:**

$$\text{EPR}_* > 1$$

- **Empirically observed operational stability floor:**

$$\text{EPR} \geq 0.25$$

The former is worst-case theoretical safety.

The latter reflects practical boundedness in realistic recursive strike systems.

5. Delay, Coupling, and Distributed Autonomy: Heuristic Extensions

Autonomous military systems rarely operate as isolated, memoryless feedback loops. Real architectures include communication latency, distributed coordination, and multi-agent policy interaction. These features modify effective amplification and corrective dominance. The analysis that follows is heuristic: the inequalities presented are analytically motivated but are not derived from a complete Lyapunov or Lyapunov–Krasovskii argument. Formal proof of the delay extension would require a Lyapunov–Krasovskii functional; formal proof of the coupling extension would require spectral analysis of the interconnection matrix. Both are outside the scope of this paper. The results below are labeled as heuristic propositions and should not be interpreted as theorems.

5.1 Corrective Delay — Proposition (Heuristic Extension)

In practice, corrective authority may act on delayed state information. Consider

$$dx_t = \rho f_{\text{rec}}(x_t)dt - \kappa_c P_t M_t g(x_{t-\tau})dt + \sigma(x_t)dW_t$$

with delay $\tau > 0$.

Using the identity

$$x(t) = x(t - \tau) + \int_{t-\tau}^t \dot{x}_s ds,$$

second-moment bounds introduce amplification inflation proportional to delay length. Under standard small-delay approximations,

$$\text{EPR}_* > 1 + \mathcal{O}(\tau)$$

is sufficient for stability.

Interpretation:

Latency increases required corrective dominance.

High-speed strike loops with delayed oversight require stronger damping to preserve boundedness.

5.2 Distributed Coupling — Proposition (Heuristic Extension)

Modern strike systems often coordinate across multiple agents. Let

$$x_t = \begin{bmatrix} x_t^{(1)} \\ \vdots \\ x_t^{(m)} \end{bmatrix}$$

with interconnection matrix C .

Coupled dynamics introduce effective amplification:

$$\rho_{\text{eff}} \approx \rho \cdot \rho(C)$$

where $\rho(C)$ is the spectral radius.

Stability requires

$$\text{EPR}_* > \rho(C).$$

If $\rho(C) > 1$, recursive escalation may propagate across agents even when individual units are locally stable.

Interpretation:

Distributed autonomy can amplify escalation beyond local policy dynamics.

5.3 Combined Effect — Analytically Motivated Bound (Unproven)

With delay and coupling:

$$\text{EPR}_* > \rho(C)(1 + \alpha\tau)$$

for some constant α depending on bandwidth.

Thus:

- Faster loops \rightarrow less delay inflation
 - Stronger coupling \rightarrow higher required damping
 - Dense multi-agent strike swarms require stricter dominance ratios
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6. Operational Implications for Autonomous Strike Architectures

The Recursive Strike Problem is not an abstract control phenomenon. It maps directly to military AI deployment design.

6.1 Escalation Stability as a Structural Constraint

Recursive strike instability manifests as:

- Expanding target sets without proportional triggers
- Escalating engagement intensity under noisy threat signals
- Reinforcement-driven overcommitment
- Positive feedback across distributed systems

Structural stability prevents these phenomena from compounding.

Bounded recursion ensures escalation cannot self-amplify indefinitely under stochastic perturbations.

6.2 Instrumenting Effective Policy Restraint

The EPR ratio is interpretable operationally:

$$\text{EPR}(t) = \frac{\text{Corrective Authority}}{\text{Recursive Amplification}}$$

Operational proxies may include:

- Constraint weight relative to learning rate

- Oversight override gain relative to adaptive update magnitude
- Mission doctrine enforcement intensity relative to policy reinforcement gain

Monitoring EPR in real time enables:

- Escalation threshold alarms
- Adaptive constraint strengthening
- Strike-loop throttling

6.3 Dominance Interpretation

The structural result yields a single escalation stability requirement:

$$\kappa_c P_t M_t > \rho \eta_b$$

Equivalently,

$$EPR_* > 1.$$

This inequality represents dominance of corrective authority over recursive amplification.

In operational terms, this requires:

- Corrective oversight intensity must exceed adaptive reinforcement gain.
- Constraint enforcement strength must not collapse below amplification levels.
- Recursive update rates must not exceed damping bandwidth.

Temporary violation of this dominance condition introduces instability risk. Sustained dominance preserves bounded escalation dynamics.

The sufficient condition is structural and uniform.

6.4 Implications for Delegated Lethal Authority

Delegated lethal autonomy is structurally safe only if corrective dominance is preserved continuously.

Temporary collapse of $\kappa_c P_t M_t$ below amplification levels introduces instability risk.

Design principles:

- Avoid recursive update rates that exceed damping bandwidth
 - Maintain lower bounds on constraint enforcement intensity
 - Limit coupling amplification across distributed agents
 - Restrict delay in supervisory intervention loops
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7. Limitations and Strategic Scope

This paper establishes a structural stability invariant. It does not solve the broader governance problem of autonomous weapons.

7.1 What This Paper Does Not Claim

It does not establish:

- Ethical alignment
- Legal compliance
- Proportionality evaluation
- Civilian harm avoidance
- Adversarial robustness
- Mission correctness

Structural stability is a necessary but insufficient condition for responsible autonomy.

7.2 Model-Class Scope

The stability result depends on the assumptions stated in Sections 2 and 3:

- Sector-bounded recursive amplification
- Coercive corrective damping
- Uniformly bounded diffusion
- Uniform lower bound on corrective authority

The sufficient condition guarantees mean-square boundedness within this model class.

Systems that violate these assumptions — including superlinear reinforcement, state-dependent explosive diffusion, adversarial injection of gain, or discontinuous update dynamics — require separate analysis.

The result should therefore be interpreted as a structural invariant under bounded recursive dynamics, not a universal property of all autonomous military systems.

7.3 Heavy-Tailed and Adversarial Regimes

The diffusion model assumes bounded second moment.

Adversarial shock regimes, discontinuous updates, or heavy-tailed disturbances may invalidate mean-square bounds.

Further work is required to extend analysis to:

- Jump-diffusion processes
 - Adversarial policy injection
 - Strategic deception environments
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7.4 Stability Is Not Strategic Wisdom

Bounded recursion prevents runaway escalation.

It does not ensure strategic prudence.

An autonomous system may remain stable yet make systematically flawed decisions.

Structural stability is a control property — not a substitute for doctrine or oversight policy.

8. Conclusion

Autonomous military systems increasingly embed recursive policy adaptation within high-speed strike loops. When such systems are granted delegated lethal authority, recursive amplification dynamics become structurally coupled to escalation risk.

This paper formalized the Recursive Strike Problem: the structural condition under which recursive autonomous strike systems remain mean-square bounded under stochastic perturbation.

Using an Itô–Lyapunov framework, we derived a sufficient dominance condition:

$$EPR_* > 1$$

guaranteeing structural stability under the defined sector bounds.

The central result is structural:

Recursive lethal autonomy remains stable only when corrective authority continuously dominates recursive amplification.

This principle applies across platforms and architectures. It provides a mathematically grounded escalation constraint for delegated strike systems operating under uncertainty.

Structural stability does not ensure ethical correctness, legal compliance, or strategic wisdom. It ensures that recursive escalation cannot compound indefinitely under stochastic feedback.

In delegated lethal architectures, that distinction is foundational.

9.1 Formal Stability Bound (REPLACEMENT)

Under the dynamics:

$$dx_t = (\rho f_{\text{rec}}(x_t) - \kappa_c P_t M_t g(x_t)) dt + \sigma(x_t) dW_t,$$

and assumptions (A1)–(A4),

the system satisfies:

$$\begin{aligned} E[|x_t|^2] \\ \leq E[|x_0|^2] e^{-2\lambda_{\min} t} \end{aligned}$$

- $\sigma_0^2 / (2\lambda_{\min})$,

with

$$\lambda_{\min} = \inf_{t \geq 0} (\kappa_c P_t M_t - \rho \eta_b).$$

Hence:

$$\sup_{\{t \geq 0\}} E[|x_t|^2] < \infty,$$

and the ultimate second-moment bound scales as:

$$\sigma_0^2 / (2 \eta_b \rho (EPR_* - 1)).$$

Structural stability holds under the sufficient condition:

$$EPR_* > 1.$$

This is a sufficiency result, not a biconditional. The necessity direction — that $EPR_* \leq 1$ implies instability — is not established here and would require a separate construction, such as exhibiting an explicit trajectory with unbounded second moment under the given sector conditions. The boundary case $EPR_* = 1$ corresponds to $\lambda_{\min} = 0$, at which the dissipation margin vanishes; mean-square boundedness at this boundary is indeterminate under the stated sector bounds and depends on sub-leading structure not captured by the present Lyapunov function. Establishing instability for $EPR_* < 1$ requires additional analysis and lies outside the scope of this paper. The result presented here guarantees that $EPR_* > 1$ is sufficient for structural stability under assumptions (A1)–(A4); no stronger claim is made.

9.2 Simulation Methodology

To evaluate practical stability transition:

- Three nonlinear recursive families were implemented:
 - Smooth saturation reinforcement
 - Stiff amplification with bounded correction
 - Switching recursive regimes with dwell-time constraint
- Amplification rates, corrective gains, delay parameters, noise magnitudes, and coupling strengths were swept over log-spaced grids.
- Over 500,000 configurations were simulated.
- Each configuration used multiple stochastic realizations.
- Stability criterion: bounded second moment over finite horizon.

Transition estimation:

- Logistic regression fit to boundedness probability.

- Bootstrap confidence intervals (10,000 resamples).

Observed transition center:

$\mathrm{EPR} \approx 0.25$

with 95% CI approximately

$[[0.24, 0.26].]$

9.3 Interpretation of the 0.25 Floor

The empirical floor arises from:

- Saturating nonlinear reinforcement
- Finite recursion bandwidth
- Implicit gain contraction
- Bounded corrective saturation

It does not contradict the sufficient theoretical condition.

It reflects non-worst-case structural dynamics.

Systems designed near the empirical floor exhibit boundedness in modeled regimes but lack theoretical worst-case margin.

9.4 Future Extensions

Future work should examine:

- Jump-diffusion escalation shocks
- Adversarial policy injection
- Heavy-tailed disturbance models
- Hybrid discrete-continuous update architectures
- Formal delay margin quantification
- Structured uncertainty and μ -analysis

The Recursive Strike Problem is a structural control problem.

Its extension to adversarial military environments requires further formalization.

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